frontiers in Physics

PERSPECTIVE
published: 06 February 2018
doi: 10.3389/fphy.2018.00004

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Is the Quilted Multiverse Consistent with a Thermodynamic Arrow of Time?

Yakir Aharonov [1,2], Eliahu Cohen [3] and Tomer Shushi [4]*

[1] School of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel, [2] Schmid College of Science, Chapman University, Orange, CA, United States, [3] H.H. Wills Physics Laboratory, University of Bristol, Bristol, United Kingdom, [4] Department of Physics, Ben-Gurion University of the Negev, Beersheba, Israel

Theoretical achievements, as well as much controversy, surround multiverse theory. Various types of multiverses, with an increasing amount of complexity, were suggested and thoroughly discussed in literature by now. While these types are very different, they all share the same basic idea: our physical reality consists of more than just one universe. Each universe within a possibly huge multiverse might be slightly or even very different from the others. The quilted multiverse is one of these types, whose uniqueness arises from the postulate that every possible event will occur infinitely many times in infinitely many universes. In this paper we show that the quilted multiverse is not self-consistent due to the instability of entropy decrease under small perturbations. We therefore propose a modified version of the quilted multiverse which might overcome this shortcoming. It includes only those universes where the minimal entropy occurs at the same instant of (cosmological) time. Only these universes whose initial conditions are fine-tuned within a small phase-space region would evolve consistently to form their "close" states at present. A final boundary condition on the multiverse may further lower the amount of possible, consistent universes. Finally, some related observations regarding the many-worlds interpretation of quantum mechanics and the emergence of classicality are discussed.

Keywords: quantum cosmology, multiverse theory, arrow of time, many-worlds interpretation, stability
PACS numbers: 05.70.-a, 98.80.Qc, 03.65.Ta

OPEN ACCESS

Edited by:
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****Correspondence:***
Eliahu Cohen
eli17c@gmail.com

Specialty section:
This article was submitted to Interdisciplinary Physics, a section of the journal Frontiers in Physics

Received: *03 November 2017*
Accepted: *19 January 2018*
Published: *06 February 2018*

Citation:
Aharonov Y, Cohen E and Shushi T (2018) Is the Quilted Multiverse Consistent with a Thermodynamic Arrow of Time? Front. Phys. 6:4. doi: 10.3389/fphy.2018.00004

1. INTRODUCTION

Multiverse theory (also known as Meta universe theory) is a group of models assuming that our physical reality encompasses more than one universe, i.e., there exists at least one more universe other than ours. Several types of such multiverses are known in literature [1–12].

Some of these models suggest that our physical reality comprises of infinitely many universes[1], while others postulate that we live in a multiverse with a finite number of universes. Most multiverse theories imply that universes might not be uniquely identified through their macroscopic state at present or past, i.e., their macrostates could be quite similar during long and even infinite time intervals. The ultimate multiverse model (also known as the mathematical multiverse) [13] satisfies this property and postulates that every possible state is in one-to-one correspondence with each universe from the multiverse horizon.

[1] In the context of this work we shall assume a discrete phase-space, meaning that all infinities are countable.

One of the most common explanations of the big-bang is given by quantum fluctuation theory, which suggests that our universe began from a quantum fluctuation, and if so, it is natural to deduce that in our physical reality these fluctuations are taking place in all of our space and time dimensions (see [8] for instance). Therefore, an infinite number of such fluctuations implies a vast multiverse of infinite number of universes.

The multiverse type that we shall focus on is the quilted multiverse [11], whose infinite space and time dimensions presumably contain infinite number of universes. In Greene's words [11]: "At any moment in time, the expanse of space contains an infinite number of separate realms-constituents of what I'll call the Quilted Multiverse-with our observable universe, all we see in the vast night sky, being but one member. Canvassing this infinite collection of separate realms, we find that particle arrangements necessarily repeat infinitely many times. The reality that holds in any given universe, including ours, is thus replicated in an infinite number of other universes across the Quilted Multiverse."

The quilted multiverse provides a theoretical probabilistic approach for the existence of events before the event horizon in our physical reality. Within the quilted multiverse, the event horizon includes events that occur infinitely many times, duplicated in infinitely many universes, which might be finite or infinite. From the characterization above we deduce that there are universes within the quilted multiverse that are not only "close" at a given time (e.g., at present), that is, similar in a sense that will be defined below, but have been very "close" for a substantially large time interval. In terms of Tegmark's hierarchy [12], the quilted multiverse we shall study corresponds to a level 1 multiverse. This type of multiverse postulates that every universe in the multiverse shares the same physical constants (e.g., the Planck constant $\hbar$ and the speed of light c), while other types of multiverses suggest that the physical constants and even physical laws are different within different universes (e.g., string theory landscape [14]). The main argument for this kind of multiverse with different physical constants is that for different universe we would have different spontaneous symmetry breaking and thus different physics. Since there are already several arguments against this type of multiverse (see [15] for instance), we will focus in this paper on the quilted multiverse where parallel universes share the same physical constants and same physical laws. We also emphasize that the quilted multiverse differs from the inflationary multiverse. The former emerges if the extent of space is infinite, while the latter's variety emerges from eternal inflationary expansion. We would assume that the multiple universes within the quilted multiverse can be coarse-grained in a countable manner, they have the same common cosmological features and same local laws, and they do not interact with each other.

We claim in this work (based on our preprint [16]) that the quilted multiverse is not consistent with basic thermodynamic assumptions. In the following section we discuss a thermodynamic arrow of time defined by the stability of entropy increase. In section 3 we present an inconsistency of the quilted multiverse and the proposed thermodynamic arrow of time. Section 4 discusses an upper bound to the number of parallel universes in the quilted multiverse, and section 5 attempts to broaden these results toward other kinds of multiverse when adding to the analysis a final boundary condition. Section 6 concludes the paper.

2. A SUBTLE THERMODYNAMIC ARROW OF TIME

Time seems to incessantly "flow" in one direction, raising the ancient question: Why? This intensively discussed question can be answered in several ways by introducing seemingly different time arrows: thermodynamic, cosmological, gravitational, radiative, particle physics (weak), quantum, and others [17, 18]. We employ in this paper the cosmological arrow of time, which points in the direction of the universe's expansion. This choice implies that parallel universes with the same macrostate will have the same time. Our main argument, however, will rely on thermodynamic stability under small perturbations which allows to define another crucial arrow of time— *the macroscopic behavior of a large system is stable against perturbations as far as its future is concerned, but for most cases is very unstable as far as its past is concerned.* [19, 20]. The positive direction of time is thus determined according the system's stability under small perturbations. Indeed, performing a slight microscopic change (not to mention a large macroscopic change) in the system's past will not change, in general, its macrostate at future times, i.e., the system will end up its time evolution with the same high entropy macrostate. However, when propagating backwards in time, such a slight change in the system's future will have far-reaching consequences on its past [19, 20]. This is the key observation we shall utilize next, akin to the thermodynamic arrow of time which relays on the second law of thermodynamic (although some subtle challenges are known in literature [21, 22]). The difference in terms of stability between future and past stems from the fact that any perturbation of a microstate will tend to make it more typical of its macrostate and thus small perturbations will not interfere with (forward in time) typical evolution. Backwards in time, however, the microstate will propagate toward a smaller phase space volume which is untypical of the macrostate. This difference in Lyapunov stability was rigorously quantified, e.g., in Hoover and Posch [23] and Sarman et al. [24].

In this perspective paper we will formally treat a universe parallel to ours, having at present time a similar macrostate or even the same macrostate, yet with a slightly different microstate as a perturbation. Then we will try to apply the above thermodynamic reasoning.

3. INCONSISTENCY OF THE QUILTED MULTIVERSE

Before we claim that the quilted multiverse is inconsistent with the instability of entropy decrease discussed in section 2, let us define some mathematical symbols which will be useful later on. First, suppose that we have an infinite (yet countable) number of universes, denoted by

$$\mathcal{U} = \{\mathcal{U}_1, \mathcal{U}_2, ...\}, \quad (1)$$

where each universe $\mathcal{U}_j$ has the (quantum) microstate $\Psi_j(t)$, and t is the cosmological time.

Further, let us define in phase space a distance measure Δ, which quantifies the difference between the microstate of the $j-$th universe, $\Psi_j(t)$, and the microstate of the $i-$th universe, $\Psi_i(t)$, at some time $t_i = t_j = t$

$$\Delta(\Psi_j(t), \Psi_i(t)) > 0, \tag{2}$$

for $i \neq j$. We hereby define $0 \leq \Delta \leq 1$ to be the ratio between the number of particles whose (possibly entangled) states are orthogonal and the total number of particles. This definition might not be unique (or the most robust) but it captures our intuition as to microscopic proximity of similar/identical macroscopic states. According to this definition and Greene's description of the quilted multiverse, for every $\varepsilon > 0$ there exist at least two universes such that

$$\Delta(\Psi_j(t), \Psi_i(t)) \leq \varepsilon, \ \forall t \in T, \tag{3}$$

where $T = [t_0, t_f]$ is some long time interval comparable with the age of the universes.

Moreover, from the above description of the quilted multiverse, we deduce that every possible event will occur an infinite (countable) number of times. Therefore, this model suggests that there should exist a set W such that

$$W = \{(i,j)|\Delta(\Psi_j(t), \Psi_i(t)) \leq \varepsilon, \forall t \in T\}, |W| = \aleph_0 \tag{4}$$

where ε is some threshold below which we may say that the universes are "close", and W is the set of all possible pairs of universes from the infinite multiverse (Equation 1) that are "close" for a period of time comparable with their age.

We now show that Equation (4) is not consistent with the thermodynamic arrow of time defined in section 2 (it will be implicitly assumed that the universe is in a non-equilibrium state). First, notice that if we have a thermodynamic system $\mathcal{V}$ with the macroscopic state $M_{\mathcal{V}}(t)$ at the cosmological time t, there exist more than one possible quantum state $\Psi_{\mathcal{V}}(t_0)$ (for every $0 < t_0 < t$) that will produce $M_{\mathcal{V}}(t)$ in time t, i.e., there is some volume in the past phase space of (quantum) microscopic states that could reproduce the present macroscopic state. However, a slightly different universe at present (analogous to a small perturbation of the first) would correspond in general to a very different volume in the past phase space [19, 20], which would in turn correspond to a markedly different macroscopic state for all times. Therefore, the backwards evolution in time (presumably dictated by the same dynamical rules) of two very close universes at present will result in two very far universes at the past (see **Figure 1**), thus negating (Equation 4).

There is only a negligible probability that two close universes at present, will evolve backwards in time to two close universes in the past (see also [25]).

Also, it is inconsistent to assume that any arbitrary change to our current universe is a valid parallel universe having the same historical source in phase space or having the same point in time of minimum entropy.

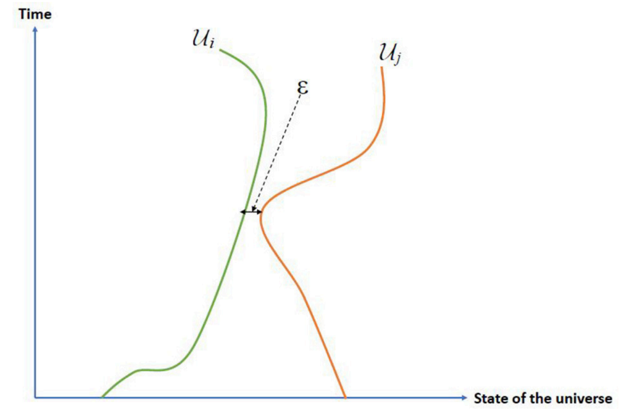

FIGURE 1 | Two "close" universes at present time were most likely "far" in the past.

The number of possible universes can be represented by the Boltzmann relation between entropy S and the set Ω of possible microstates corresponding to the same macrostate,

$$S = k_B \ln |\Omega|, \tag{5}$$

where k_B is the Boltzmann constant.

Then, given the entropy of the i-th universe, $S_{\mathcal{U}_i}$, $|\Omega_{\mathcal{U}_i}|$ is

$$|\Omega_{\mathcal{U}_i}| = e^{S_{\mathcal{U}_i}/k_B}. \tag{6}$$

Assuming that during its 13.8 billion years of history the universe has reached a very large entropy $S_{\mathcal{U}_i} \gg k_B$, we have a huge set of possible microstates $\Omega_{\mathcal{U}_i}$. Let us examine a pair of universes having at t' close states, i.e., $\Delta(\Psi_i(t'), \Psi_j(t')) \simeq 0$, where $\Psi_i \in \Omega_{\mathcal{U}_i}$, $\Psi_j \in \Omega_{\mathcal{U}_j}$ and $|\Omega_{\mathcal{U}_i}| = |\Omega_{\mathcal{U}_j}|$. We claim that at arbitrary time $t'' \ll t'$ they will most likely have $|\Omega_{\mathcal{U}_i}| \neq |\Omega_{\mathcal{U}_j}|$, and the probability that $\Delta(\Psi_i(t''), \Psi_j(t'')) \simeq 0$ will be close to zero. This follows from the fact that $\Omega_{\mathcal{U}_k}(t'')$ is now the backward-in-time evolution of $\Omega_{\mathcal{U}k}(t')$, $k = i, j$, which is a set with very large number of possibilities, so that the probability

$$\Pr\left(|\Omega_{\mathcal{U}_i}(t'')| \simeq |\Omega_{\mathcal{U}_j}(t'')| \mid |\Omega_{\mathcal{U}_i}(t')| \simeq |\Omega_{\mathcal{U}_j}(t')|\right), \tag{7}$$

will be zero.

Another way to see this inconsistency is to consider the point of minimal entropy during the lifetime of our universe. When picking at random another hypothetical universe having at present the same macrostate as our universe, it is most likely to have its minimal entropy at some other time different from ours (most likely after ours). Hence, the histories of the two universes cannot be the same, unless we focus at present only on the zero measure of macrostates having their minimal entropy at exactly the same time as ours.

4. UPPER BOUND TO THE NUMBER OF PARALLEL UNIVERSES IN THE QUILTED MULTIVERSE

We shall try to approach the problem from a different perspective now, beginning with some qualitative considerations. One should note two extreme distance scales between universes in a multiverse. When two universes are extremely close (that is, different but virtually indistinguishable so that $0 < \Delta \ll 1$) at some point in time, they may have a non-negligible probability evolving backwards to extremely close initial states, thereby creating no inconsistency. However, having infinitely many universes which are identical to ours for all practical purposes is not too interesting. On the other hand, if two universes are far apart right now, stability (which corresponds to small perturbation) again plays no role. But this is not the case we wish to rule out.

Between these two contingent cases, lie the problematic distances to which instability considerations can be applied. This may pose a constraint on the distribution of universes within a multiverse—there might be infinitely many universes which are very far from each other and an infinite number of universes which are extremely close, but we do not expect too many universes to be intermediately close when we demand consistency over long times.

Let us examine now for concreteness a $6N$-dimensional quantum phase space. Let us suppose for simplicity that the phase space is discrete and focus on some large yet finite part of it. We can therefore think about this sector as a hypercube with a fine grid. A universe j within this sector is represented e.g., by a Wigner quasi-distribution $W\left(X_j, P_j\right)$ on the grid, where X_j/P_j encapsulates the three position/momentum vectors, respectively, of each particle in this universe. We now start to gradually fill the hypercube with more and more distributions. We begin with those having a slight overlap (or no overlap at all) with the original one and with each other, thus corresponding to universes which are very different. As this process continues, we will have to fill the finite phase space with more and more distributions, closer to each other, until a point (let us denote it by $\Delta = D$) when they become very close, such that distance between the universes characterizes a small perturbation. We would thus unavoidably create at least two universes that are too close to each other. Too close, in the sense that one can be thought of as a small perturbation to the other, and then upon backward evolution in time, they would most likely reach inconsistent states.

We now apply similar arguments to those appearing at the end of the previous section. It seems that in a countably infinite phase space (allowing a countably infinite number of parallel universes) and a finite point in time t, there might be only a finite number of consistent parallel universes whose Δ separation is very close until time $t = 0$, but we leave this as a conjecture. In any case, we would like to point out that an infinite number of parallel universes might be ruled out this way just as a result of thermodynamic considerations.

To resolve this apparent shortcoming of the quilted multiverse we must pose a condition on the possible distance between the universes, and eventually on their density. In case that

$$\Delta\left(\Psi_i, \Psi_j\right) \geq D,\ \forall\, i, j \tag{8}$$

for some threshold $0 < D < 1$, we potentially find a consistent multiverse that does not violate the aforementioned notion of stability. To this microscopic condition we add the macroscopic demand that despite the distance, the various universe would still describe the same macrostate at all times, and in particular would have their minimal entropy state at the same cosmological time. Of course, this multiverse is different from the quilted multiverse, and hence we call it the "Modified quilted multiverse."As opposed to the ordinary quilted multiverse, it might coexist with thermodynamic laws, yet may still violate other basic requirements like Occam's razor[2].

5. GENERALIZATIONS EMPLOYING A FINAL BOUNDARY CONDITION

It could be interesting to apply the above considerations to other kinds of multiverses. However, when the values of physical constants, and moreover, physical laws themselves, in other universes become different from those we know now in our universe, the distance between our universe and others might be very large at present (and furthermore vary with time). Therefore, it is not obvious how to apply stability considerations to these kinds of multiverse.

On the other side of the multiverse scale, there is the many worlds interpretation of quantum mechanics (also known as the quantum multiverse). In previous works [26–28], two of us have employed a final boundary condition on the universe which is of special kind. This unique boundary condition allowed us to overcome some conceptual difficulties appearing in the many worlds interpretation. In particular, we suggested a model for a macroscopically reversible universe without the need of employing infinitely many parallel universes. Furthermore, we were able to devise an effective collapse mechanism in this single-branched "modest" multiverse structure. Finally, our proposed two-time decoherence scheme allowed to draw the boundary between the classical and quantum regimes.

These past results hint that the multitude of universes proposed by the many-worlds interpretation may not be needed in order to account for our empirical observations in a time-symmetric manner. Other kinds of multiverse can be handled the same way, and indeed, posing both initial and final boundary conditions on a multiverse should dramatically lower the measure of possible universes within it: Regardless of the dynamics, when the final state of the multiverse is evolved backwards in time, it must be compatible with any earlier state. As noted in Aharonov and Reznik B [29], some final boundary conditions give rise to the Born rule, and are hence preferable

[2]In the quilted multltiverse, the number (or commonness) of universes does not correspond to probability/Born rule, but in contrast, for the many-worlds-type multiverse we would have to employ a different logic as presented in section 5.

over others. Further conditions on the final state may even isolate a unique set of final boundary conditions with a higher explanatory power. These include our proposal for a quantum universe having a natural notion of classicality emerging from the requirement to store microscopic information in a redundant manner [26–28]. A recently analyzed feature of this time-symmetric universe is a top-down logical structure [30], which could further shed light on the subtle relations between micro and macro scales.

6. CONCLUSIONS

Multiverse theory has various models that describe different structures of the physical reality. One of these models is the quilted multiverse, which postulates that every possible event is occurring infinitely many times in nature, thus there are infinitely many universes resembling ours. At first glance, this model seems to be self-consistent. However, we have shown that this model negates basic thermodynamic principles. The difference between microstates in two "close" universes cannot be ϵ small at each point in time, or even along a finite, sufficiently large time interval. Therefore, any possible type of multiverse would better not assume such a relation between two universes. Moreover, every universe must have its unique past and future in the sense that there is no other universe with the same, or even very close, state over a substantial part of its life time. We therefore have to limit ourselves only to those universes whose macrostates at present time evolve backwards to the same point in time of minimal entropy such as ours. These obviously reside in a very small fraction of phase space and may evolve in a consistent way. Further constraints on the number of possible universes may arise when augmenting this analysis with a final boundary condition on the multiverse.

These findings corroborate previous ones of our group [26–28], suggesting that in addition to apparent inconsistencies and various conceptual problems, the overwhelming multitude exhibited by multiverse theory in general, and the quilted mutliverse/many worlds interpretation in particular, might not be needed in order to satisfactorily account for our observations in the classical and quantum realms using a single, unique universe.

AUTHOR CONTRIBUTIONS

YA: Initiated the work; YA, EC, and TS: Developed the presented ideas; EC and TS: Wrote the manuscript with comments from YA.

ACKNOWLEDGMENTS

We wish to thank Avshalom C. Elitzur and Daniel Rohrlich for helpful comments. YA acknowledges support of the Israel Science Foundation Grant No. 1311/14, of the ICORE Excellence Center "Circle of Light" and of DIP, the German-Israeli Project cooperation. EC was supported by ERC AdG NLST. TS thanks the John Templeton Foundation (Project ID 43297) and from the Israel Science Foundation (grant no. 1190/13). The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of any of these supporting foundations. A earlier version of this work is available as an arXiv preprint [16] (under arXiv.org perpetual, non-exclusive license to distribute this article).

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Conflict of Interest Statement: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.